# Singularities of 3-parameter line congruences in $\mathbb{R}^{4}$ 

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#### Abstract

In this talk we define 3-parameter line congruences in $\mathbb{R}^{4}$, which is nothing but a 3parameter family of lines over a hypersurface in $\mathbb{R}^{4}$. Locally, we write $\mathcal{C}=\{x(u), \xi(u)\}$, where $x: U \rightarrow \mathbb{R}^{4}$ (reference hypersurface) and $\xi: U \rightarrow \mathbb{R}^{4} \backslash\{0\}$ (director hypersurface) are smooth, where $U \subset \mathbb{R}^{3}$ is open. We also investigate the singularities associated to a special class of congruences, called Blaschke affine normal congruences, which occurs when the reference hypersurface $x$ is non-degenerate and the director hypersurface $\xi$ is given by its Blaschke normal vector field. Our goal is to show that the generic singularities of


$$
\begin{align*}
F_{(x, \xi)}: U \times I & \rightarrow \mathbb{R}^{4}  \tag{0.1}\\
(u, t) & \mapsto x(u)+t \xi(u)
\end{align*}
$$

where $I$ is an open interval, are the Lagrangian stable singularities, providing a positive answer to the conjecture presented in [1].

Joint work with: Débora Lopes (UFS) and Maria Aparecida Soares Ruas (ICMC).

## References

[1] Izumiya, S., Saji, K., Takeuchi, N. Singularities of line congruences . Proc. of the Royal Society of Edinb., 133A, 1341-1359, 2003.
[2] Lopes, D., Ruas, M. A. S., Santos, I. C. . Singularities of 3-parameter line congruences in $\mathbb{R}^{4}$. arXiv:2110.10818v2.

