## Singularities of 3-parameter line congruences in $\mathbb{R}^4$

## Igor Chagas Santos

Instituto de Ciências Matemáticas e de Computação - USP igor.chs34@usp.br

**Abstract:** In this talk we define 3-parameter line congruences in  $\mathbb{R}^4$ , which is nothing but a 3parameter family of lines over a hypersurface in  $\mathbb{R}^4$ . Locally, we write  $\mathcal{C} = \{x(u), \xi(u)\}$ , where  $x : U \to \mathbb{R}^4$  (reference hypersurface) and  $\xi : U \to \mathbb{R}^4 \setminus \{0\}$  (director hypersurface) are smooth, where  $U \subset \mathbb{R}^3$  is open. We also investigate the singularities associated to a special class of congruences, called Blaschke affine normal congruences, which occurs when the reference hypersurface x is non-degenerate and the director hypersurface  $\xi$  is given by its Blaschke normal vector field. Our goal is to show that the generic singularities of

$$F_{(x,\xi)}: U \times I \to \mathbb{R}^4$$

$$(u,t) \mapsto x(u) + t\xi(u),$$

$$(0.1)$$

where I is an open interval, are the Lagrangian stable singularities, providing a positive answer to the conjecture presented in [1].

Joint work with: Débora Lopes (UFS) and Maria Aparecida Soares Ruas (ICMC).

## References

- Izumiya, S., Saji, K., Takeuchi, N. Singularities of line congruences. Proc. of the Royal Society of Edinb., 133A, 1341-1359, 2003.
- [2] Lopes, D., Ruas, M. A. S., Santos, I. C. . Singularities of 3-parameter line congruences in  $\mathbb{R}^4$ . arXiv:2110.10818v2.